

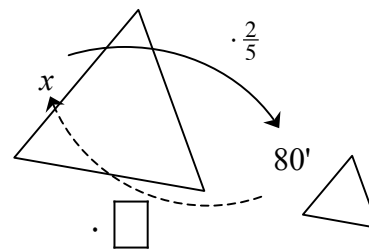
SCALING TO SOLVE PERCENT AND OTHER PROBLEMS 6.2.4 – 6.2.6

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Recall that a reciprocal is the multiplicative inverse of a number, that is, the product of the two numbers is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, $\frac{1}{2}$ is $\frac{2}{1}$, and 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by multiplying by $(1 + \text{the percent})$ and decreasing by a certain percent may be found by multiplying by $(1 - \text{the percent})$.

Example 1

The large triangle at right was reduced by a scale factor of $\frac{2}{5}$ to create a similar triangle. If the side labeled x now has a length of 80' in the new figure, what was the original length?



To undo the reduction, multiply 80' by the reciprocal of $\frac{2}{5}$, namely $\frac{5}{2}$, or divide 80' by $\frac{2}{5}$.

$80' \div \frac{2}{5}$ is the same as $80' \cdot \frac{5}{2}$, so $x = 200'$.

Example 2

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and a Giant One

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2} \left(\frac{2}{3}x \right) = \frac{3}{2}(12)$$

$$x = 18$$

Example 3

Samantha wants to leave a 15% tip on her lunch bill of \$12.50. What scale factor should be used and how much money should she leave?

Since tipping increases the total, the scale factor is $(1 + 15\%) = 1.15$.
She should leave $(1.15)(12.50) = \$14.38$ or about \$14.50.

Example 4

Carlos sees that all DVDs are on sales at 40% off. If the regular price of a DVD is \$24.95, what is the scale factor and how much is the sale price?

If items are reduced 40%, the scale factor is $(1 - 40\%) = 0.60$.
The sale price is $(0.60)(24.95) = \$14.97$.

Problems

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$ and the new width is 40 cm. What was the original width?
2. A side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?
3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet ($\frac{2}{75}$). If on the design, the fire pit is 0.5 inches away from the house, how far from the house, in feet, should the fire pit be dug?
4. After a very successful year, Cheap-Rentals raised salaries by a scale factor of $\frac{11}{10}$. If Luan now makes \$14.30 per hour, what did she earn before?
5. Solve: $\frac{3}{4}x = 60$
6. Solve: $\frac{2}{5}x = 42$
7. Solve: $\frac{3}{5}y = 40$
8. Solve: $-\frac{8}{3}m = 6$
9. What is the total cost of a \$39.50 family dinner after you add a 20% tip?
10. If the current cost to attend Magicland Park is now \$29.50 per person, what will be the cost after a 8% increase?
11. Winter coats are on clearance at 60% off. If the regular price is \$79, what is the sale price?
12. The company president has offered to reduce her salary 10% to cut expenses. If she now earns \$175,000, what will be her new salary?

Answers

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|------------|--------------|-------------------------|--------------------|
| 1. 16 cm | 2. 27 inches | 3. $18\frac{3}{4}$ feet | 4. \$13.00 |
| 5. 80 | 6. 105 | 7. $66\frac{2}{3}$ | 8. $-2\frac{1}{4}$ |
| 9. \$47.40 | 10. \$31.86 | 11. \$31.60 | 12. \$157,500 |